CURRICULUM, PEDAGOGY AND BEYOND

THE MATHEMATICAL ASSOCIATION OF VICTORIA

MAV24 CONFERENCE

OXFORD UNIVERSITY PRESS

Understanding division

A computational thinking approach

Alex de Lacy
Senior Learning Designer: Mathematics (Secondary)
Oxford University Press

What is an algorithm?

A step-by-step procedure for solving a problem which terminates.

Why are we talking about algorithms?

Year 5	Follow a mathematical algorithm involving branching and repetition (iteration); create and use algorithms involving a sequence of steps and decisions and digital tools to experiment with factors, multiples and divisibility; identify, interpret and describe emerging patterns (VC2M5N10)
Year 6	Design and use algorithms involving a sequence of steps and decisions that use rules to generate sets of numbers; identify, interpret and explain emerging patterns (VC2M6A03)
Year 7	Design algorithms involving a sequence of steps and decisions that will sort and classify sets of shapes according to their attributes, and describe how the algorithms work (VC2M7SP04)
Year 8	Use algorithms and related testing procedures to identify and correct errors (VC2M8A04)
	Design and test algorithms involving a sequence of steps and decisions that identify congruency or similarity of shapes, and describe how the algorithm works (VC2M8SP04)
Year 9	Design, test and refine algorithms involving a sequence of steps and decisions based on geometric constructions and theorems; discuss and evaluate refinements (VC2M9SP03)
Year 10	Implement algorithms that use data structures using pseudocode or a general purpose programming language (VC2M10A06)
Year 10A	Devise and use algorithms and simulations to solve mathematical problems (VC2M10AA02)

What skills do students need?

Follow an algorithm

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Year 8	**Use algorithms** and related testing procedures to identify and correct errors (VC2M8A04)
	Design and test algorithms involving a sequence of steps and decisions that identify congruency or similarity of shapes, and describe how the algorithm works (VC2M8SP04)
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What skills do students need?

Follow an algorithm

Represent an algorithm

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What skills do students need?

Follow an algorithm

Represent an algorithm

Design an algorithm

What skills do students need?

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What skills do students need?

Follow an algorithm

Represent an algorithm

Write an algorithm

Understand how an algorithm works

What skills do students need?

Year 5	Follow a mathematical algorithm involving branching and repetition (iteration); create and use algorithms involving a sequence of steps and decisions and digital tools to experiment with factors, multiples and divisibility; identify, interpret and describe emerging patterns (VC2M5N10)
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What algorithms do we use in maths?

Vertical addition algorithm

	1	1	
	7	5	8
+		9	3
	8	5	1

- Work from right-to-left
- Carry/regroup values
- Place value is key

Vertical subtraction algorithm

$$
\begin{array}{rccc}
 & \not{7}^{\color{red}6} & {}^{\color{red}1}5 & 8 \\
- & & 9 & 3 \\
\hline
 & 6 & 6 & 5 \\
\hline
\end{array}
$$

- Work from right-to-left
- Carry/regroup values
- Place value is key

Vertical multiplication algorithm

			1	
		1	2	
		1	3	7
×			2	3
	1	4	1	1
+	2	7	4	0
	3	1	5	1

- Work from right-to-left
- Carry/regroup values
- Place value is key
- Uses the addition algorithm

What do these algorithms have in common?

Division

Consider the following division problem:

Dividend ÷ Divisor

Quotient: the largest number which when multiplied by the divisor gives a value less than or equal to the dividend.

Remainder: the difference between the dividend and the product of the quotient and divisor.

Can the remainder be greater than the divisor?

Division

We want to write the division in the following format:

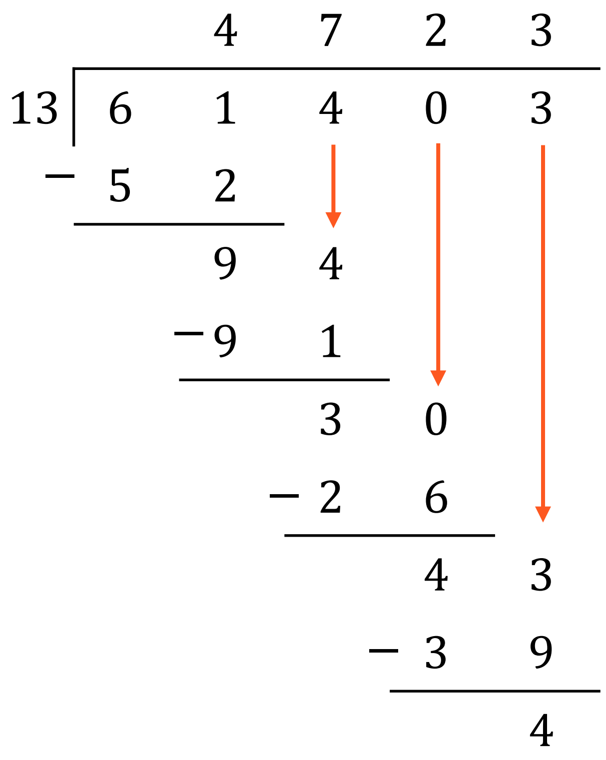
$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

Let's try $61\,403 \div 13$ using long division.

The long division algorithm

$$61\,403 \div 13$$

n	$n \times 13$
1	13
2	26
3	39
4	52
5	65
6	78
7	91
8	104
9	117
10	130



1. Write the first 10 multiples of the divisor.
2. Divide left-most digit by divisor and write quotient.
3. Multiply quotient by divisor.
4. Subtract result.
5. Bring down next value(s).
6. Repeat steps or write remainder.

What do we do in the long division algorithm?

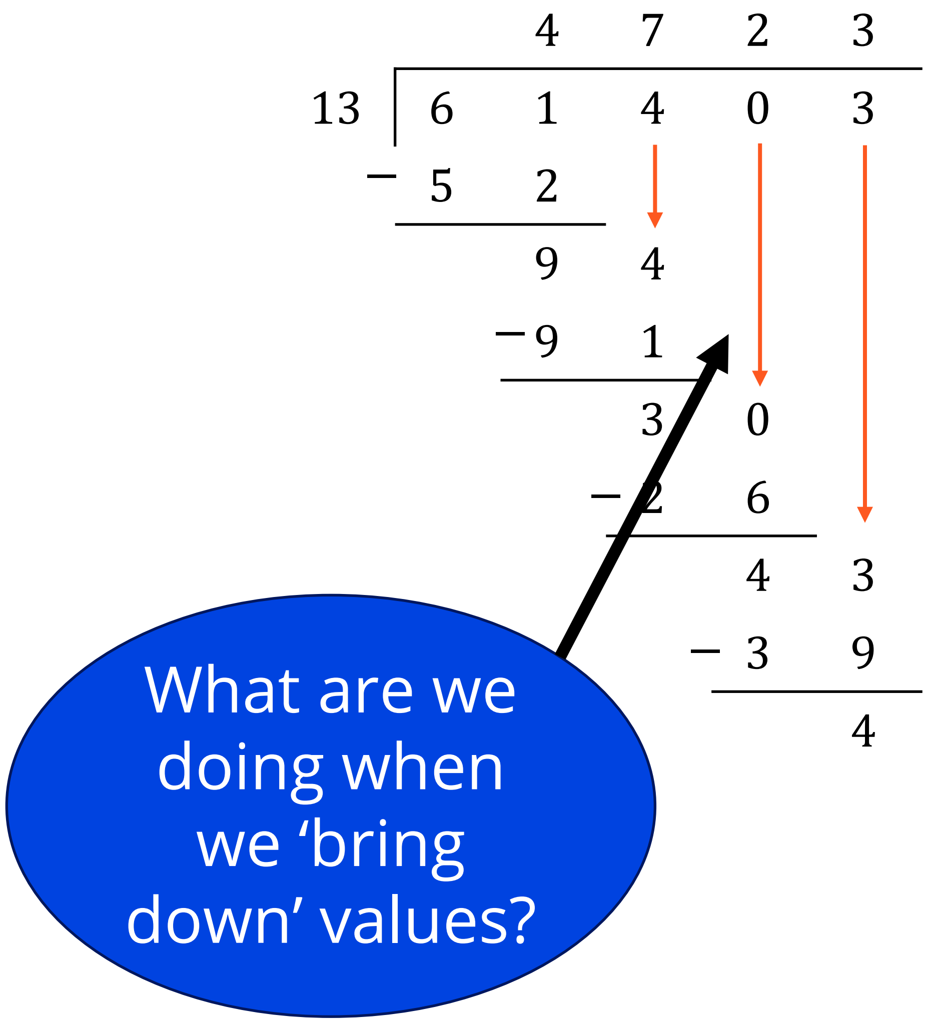
- Counting in multiples
- (Maybe) uses the multiplication algorithm
- Uses the subtraction algorithm
- Mysterious steps like ‘bring down’
- Work from left-to-right

No wonder this algorithm is confusing!

Let’s investigate!

Can we work out how the algorithm works?

n	$n \times 13$
1	13
2	26
3	39
4	52
5	65
6	78
7	91
8	104
9	117
10	130



1. Write the first 10 multiples of the divisor.
2. Divide left-most digit by divisor and write quotient.
3. Multiply quotient by divisor.
4. Subtract result.
5. Bring down next value(s).
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Can we work out how the algorithm works?

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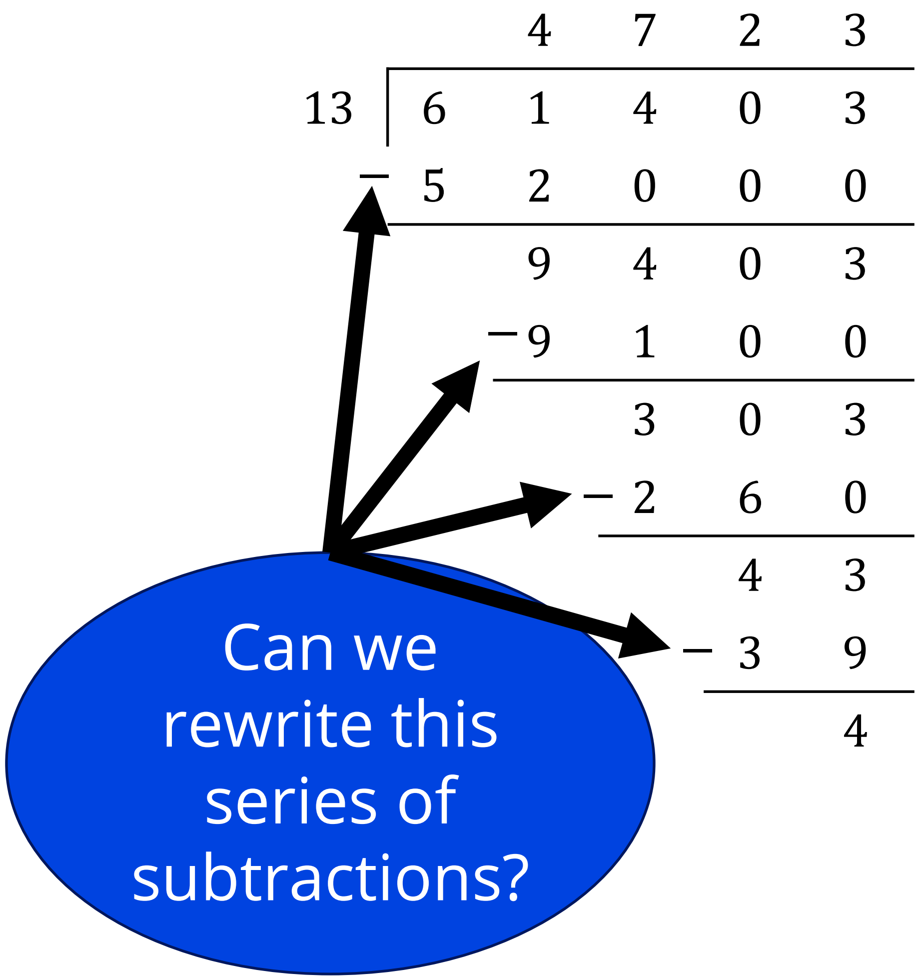
1	13
2	26
3	39
4	52
5	65
6	78
7	91
8	104
9	117
10	130



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2. Divide left-most digit by divisor and write quotient.
3. Multiply quotient by divisor.
4. Subtract result.
5. Bring down next value(s).
6. Repeat steps or write remainder.

Can we work out how the algorithm works?

$$\begin{array}{rrrrrr}
 & & 4 & 7 & 2 & 3 \\ \cline{2-6}
13 \,\big| & 6 & 1 & 4 & 0 & 3 \\
- & 5 & 2 & 0 & 0 & 0 \\ \cline{2-6}
 & & 9 & 4 & 0 & 3 \\
 & - & 9 & 1 & 0 & 0 \\ \cline{3-6}
 & & & 3 & 0 & 3 \\
 & & - & 2 & 6 & 0 \\ \cline{4-6}
 & & & & 4 & 3 \\
 & & & - & 3 & 9 \\ \cline{5-6}
 & & & & & 4
\end{array}$$

$$\begin{aligned}
61\,403 \; & - 52\,000 \\
& -9100 \\
& -260 \\
& -39 \\
& = 4
\end{aligned}$$

Can we rewrite this series of subtractions?

Can we work out how the algorithm works?

			4	7	2	3
13		6	1	4	0	3
	−	5	2	0	0	0
			9	4	0	3
			−9	1	0	0
				3	0	3
				−2	6	0
					4	3
					−3	9
						4

$$61\,403 = 52\,000 \\ +9100 \\ +260 \\ +39 \\ +4$$

What is special about these numbers?

Can we work out how the algorithm works?

$$
\begin{array}{rrrrrr}
 & & 4 & 7 & 2 & 3 \\ \cline{2-6}
13 & \big) 6 & 1 & 4 & 0 & 3 \\
 & -5 & 2 & 0 & 0 & 0 \\ \cline{2-6}
 & & 9 & 4 & 0 & 3 \\
 & & -9 & 1 & 0 & 0 \\ \cline{3-6}
 & & & 3 & 0 & 3 \\
 & & & -2 & 6 & 0 \\ \cline{4-6}
 & & & & 4 & 3 \\
 & & & & -3 & 9 \\ \cline{5-6}
 & & & & & 4
\end{array}
$$

$$
\begin{aligned}
61\,403 = {} & 13 \times 4000 \\
& +13 \times 700 \\
& +13 \times 20 \\
& +13 \times 3 \\
& +4
\end{aligned}
$$

What is special about these numbers?

Can we work out how the algorithm works?

$$
\begin{array}{r|rrrrr}
 & & 4 & 7 & 2 & 3 \\ \hline
13 & 6 & 1 & 4 & 0 & 3 \\
 & -5 & 2 & 0 & 0 & 0 \\ \hline
 & & 9 & 4 & 0 & 3 \\
 & & -9 & 1 & 0 & 0 \\ \hline
 & & & 3 & 0 & 3 \\
 & & & -2 & 6 & 0 \\ \hline
 & & & & 4 & 3 \\
 & & & & -3 & 9 \\ \hline
 & & & & & 4
\end{array}
$$

$$
\begin{aligned}
61\,403 = {} & 13 \times 4 \times 1000 \\
& +13 \times 7 \times 100 \\
& +13 \times 2 \times 10 \\
& +13 \times 3 \times 1 \\
& +4
\end{aligned}
$$

What is special about these numbers?

Can we work out how the algorithm works?

$$
\begin{array}{rrrrrr}
 & & \color{green}{4} & \color{green}{7} & \color{green}{2} & \color{green}{3} \\
\cline{2-6}
13 & \multicolumn{1}{|r}{6} & 1 & 4 & 0 & 3 \\
- & 5 & 2 & 0 & 0 & 0 \\
\cline{2-6}
 & & 9 & 4 & 0 & 3 \\
 & & -9 & 1 & 0 & 0 \\
\cline{3-6}
 & & & 3 & 0 & 3 \\
 & & & -2 & 6 & 0 \\
\cline{4-6}
 & & & & 4 & 3 \\
 & & & & -3 & 9 \\
\cline{5-6}
 & & & & & 4
\end{array}
$$

$$
\begin{aligned}
61\,403 = {} & 13 \times \color{green}{\mathbf{4}} \times 1000 \\
& +13 \times \color{green}{\mathbf{7}} \times 100 \\
& +13 \times \color{green}{\mathbf{2}} \times 10 \\
& +13 \times \color{green}{\mathbf{3}} \times 1 \\
& +4
\end{aligned}
$$

We've found the quotient!

Can we work out how the algorithm works?

$$
\begin{array}{rrrrrr}
 & & \color{green}{\mathbf{4}} & \color{green}{\mathbf{7}} & \color{green}{\mathbf{2}} & \color{green}{\mathbf{3}} \\
\cline{2-6}
13 \,\big| & 6 & 1 & 4 & 0 & 3 \\
- & 5 & 2 & 0 & 0 & 0 \\
\cline{2-6}
 & & 9 & 4 & 0 & 3 \\
 & - & 9 & 1 & 0 & 0 \\
\cline{3-6}
 & & & 3 & 0 & 3 \\
 & & - & 2 & 6 & 0 \\
\cline{4-6}
 & & & & 4 & 3 \\
 & & & - & 3 & 9 \\
\cline{5-6}
 & & & & & 4
\end{array}
$$

$$
\begin{aligned}
61\,403 = {} & 13 \times \color{green}{\mathbf{4}} \times 1000 \\
& +13 \times \color{green}{\mathbf{7}} \times 100 \\
& +13 \times \color{green}{\mathbf{2}} \times 10 \\
& +13 \times \color{green}{\mathbf{3}} \times 1 \\
& +4
\end{aligned}
$$

Let’s take out the factor of 13.

Can we work out how the algorithm works?

$$
\begin{array}{r|rrrrr}
 & & 4 & 7 & 2 & 3 \\
\hline
13 & 6 & 1 & 4 & 0 & 3 \\
- & 5 & 2 & 0 & 0 & 0 \\
\hline
 & & 9 & 4 & 0 & 3 \\
 & & -9 & 1 & 0 & 0 \\
\hline
 & & & 3 & 0 & 3 \\
 & & & -2 & 6 & 0 \\
\hline
 & & & & 4 & 3 \\
 & & & & -3 & 9 \\
\hline
 & & & & & 4
\end{array}
$$

$$
\begin{aligned}
61\,403 = 13 \times \mathbf{4} \times 1000 \\
+13 \times \mathbf{7} \times 100 \\
+13 \times \mathbf{2} \times 10 \\
+13 \times \mathbf{3} \times 1 \\
+4
\end{aligned}
$$

$$61\,403 = 13 \times (\mathbf{4} \times 1000 + \mathbf{7} \times 100 + \mathbf{2} \times 10 + \mathbf{3} \times 1) + 4$$

Can we work out how the algorithm works?

$$
\begin{array}{r|rrrrr}
 & & \mathbf{4} & \mathbf{7} & \mathbf{2} & \mathbf{3} \\ \cline{2-6}
13 & 6 & 1 & 4 & 0 & 3 \\
 & -5 & 2 & 0 & 0 & 0 \\ \cline{2-6}
 & & 9 & 4 & 0 & 3 \\
 & & -9 & 1 & 0 & 0 \\ \cline{3-6}
 & & & 3 & 0 & 3 \\
 & & & -2 & 6 & 0 \\ \cline{4-6}
 & & & & 4 & 3 \\
 & & & & -3 & 9 \\ \cline{5-6}
 & & & & & 4
\end{array}
$$

$$
\begin{aligned}
61\,403 = {} & 13 \times \mathbf{4} \times 1000 \\
& +13 \times \mathbf{7} \times 100 \\
& +13 \times \mathbf{2} \times 10 \\
& +13 \times \mathbf{3} \times 1 \\
& +4
\end{aligned}
$$

$$61\,403 = 13 \times (\mathbf{4}000 + \mathbf{7}00 + \mathbf{2}0 + \mathbf{3}) + 4$$

Can we work out how the algorithm works?

$$
\begin{array}{r|rrrrr}
 & & 4 & 7 & 2 & 3 \\ \hline
13 & 6 & 1 & 4 & 0 & 3 \\
- & 5 & 2 & 0 & 0 & 0 \\ \hline
 & & 9 & 4 & 0 & 3 \\
 & & -9 & 1 & 0 & 0 \\ \hline
 & & & 3 & 0 & 3 \\
 & & & -2 & 6 & 0 \\ \hline
 & & & & 4 & 3 \\
 & & & & -3 & 9 \\ \hline
 & & & & & 4
\end{array}
$$

$$
\begin{aligned}
61\,403 = & \; 13 \times \mathbf{4} \times 1000 \\
& +13 \times \mathbf{7} \times 100 \\
& +13 \times \mathbf{2} \times 10 \\
& +13 \times \mathbf{3} \times 1 \\
& +4
\end{aligned}
$$

$$61\,403 = 13 \times \mathbf{4723} + 4$$

$$\text{Dividend} = \text{Divisor} \times \textbf{Quotient} + \text{Remainder}$$

What do we do next?

Let’s do it again!

Let’s try 32 468 ÷ 17 using long division.

Let’s do it again!

32 468 ÷ 17

n	$n \times 17$
1	17
2	34
3	51
4	68
5	85
6	102
7	119
8	136
9	153
10	170

$$
\begin{array}{r|rrrrr}
 & & 1 & 9 & 0 & 9 \\ \cline{2-6}
17 & 3 & 2 & 4 & 6 & 8 \\
- & 1 & 7 & & & \\ \cline{2-3}
 & 1 & 5 & 4 & & \\
- & 1 & 5 & 3 & & \\ \cline{2-4}
 & & & 1 & 6 & 8 \\
 & & - & 1 & 5 & 3 \\ \cline{4-6}
 & & & & 1 & 5
\end{array}
$$

1. Write the first 10 multiples of the divisor.
2. Divide left-most digit by divisor and write quotient.
3. Multiply quotient by divisor.
4. Subtract result.
5. Bring down next value(s).
6. Repeat steps or write remainder.

Let's do it again!

$32\,468 \div 17$

n	$n \times 17$
1	17
2	34
3	51
4	68
5	85
6	102
7	119
8	136
9	153
10	170

		1	9	0	9
17	3	2	4	6	8
−	1	7	0	0	0
	1	5	4	6	8
−	1	5	3	0	0
			1	6	8
		−	1	5	3
				1	5

1. Write the first 10 multiples of the divisor.
2. Divide left-most digit by divisor and write quotient.
3. Multiply quotient by divisor.
4. Subtract result.
5. Bring down next value(s).
6. Repeat steps or write remainder.

Let's do it again!

$$
\begin{array}{r|rrrrr}
 & & 1 & 9 & 0 & 9 \\ \hline
17 & 3 & 2 & 4 & 6 & 8 \\
- & 1 & 7 & 0 & 0 & 0 \\ \cline{2-6}
 & 1 & 5 & 4 & 6 & 8 \\
- & 1 & 5 & 3 & 0 & 0 \\ \cline{2-6}
 & & & 1 & 6 & 8 \\
 & & - & 1 & 5 & 3 \\ \cline{4-6}
 & & & & 1 & 5
\end{array}
$$

$$
\begin{aligned}
32\,468 - {} & 17\,000 \\
& -15\,300 \\
& -153 \\
& = 15
\end{aligned}
$$

Let’s do it again!

$$
\begin{array}{r|rrrrr}
 & & 1 & 9 & 0 & 9 \\
\hline
17 & 3 & 2 & 4 & 6 & 8 \\
- & 1 & 7 & 0 & 0 & 0 \\
\hline
 & 1 & 5 & 4 & 6 & 8 \\
- & 1 & 5 & 3 & 0 & 0 \\
\hline
 & & & 1 & 6 & 8 \\
 & & - & 1 & 5 & 3 \\
\hline
 & & & & 1 & 5
\end{array}
$$

$$
\begin{aligned}
32\,468 = {} & 17\,000 \\
& +15\,300 \\
& +153 \\
& +15
\end{aligned}
$$

Let's do it again!

$$
\begin{array}{r|rrrrr}
 & & 1 & 9 & 0 & 9 \\
\hline
17 & 3 & 2 & 4 & 6 & 8 \\
- & 1 & 7 & 0 & 0 & 0 \\
\hline
 & 1 & 5 & 4 & 6 & 8 \\
- & 1 & 5 & 3 & 0 & 0 \\
\hline
 & & & 1 & 6 & 8 \\
 & & - & 1 & 5 & 3 \\
\hline
 & & & & 1 & 5
\end{array}
$$

$$
\begin{aligned}
32\,468 = \; & 17 \times 1000 \\
& +17 \times 900 \\
& +17 \times 9 \\
& +15
\end{aligned}
$$

Let’s do it again!

		1	9	0	9
17	3	2	4	6	8
−	1	7	0	0	0
	1	5	4	6	8
−	1	5	3	0	0
			1	6	8
		−	1	5	3
				1	5

$$
\begin{aligned}
32\,468 = \ & 17 \times 1 \times 1000 \\
& +17 \times 9 \times 100 \\
& +17 \times 9 \times 1 \\
& +15
\end{aligned}
$$

Is something missing?

Let's do it again!

$$
\begin{array}{r|rrrrr}
 & & 1 & 9 & 0 & 9 \\
\hline
17 & 3 & 2 & 4 & 6 & 8 \\
- & 1 & 7 & 0 & 0 & 0 \\
\hline
 & 1 & 5 & 4 & 6 & 8 \\
- & 1 & 5 & 3 & 0 & 0 \\
\hline
 & & & 1 & 6 & 8 \\
 & & - & 1 & 5 & 3 \\
\hline
 & & & & 1 & 5
\end{array}
$$

$$
\begin{aligned}
32\,468 = \; & 17 \times 1 \times 1000 \\
& +17 \times 9 \times 100 \\
& \mathbf{+17 \times 0 \times 10} \\
& +17 \times 9 \times 1 \\
& +15
\end{aligned}
$$

Let's do it again!

$$
\begin{array}{r|rrrrr}
 & & \mathbf{1} & \mathbf{9} & \mathbf{0} & \mathbf{9} \\
\hline
17 & 3 & 2 & 4 & 6 & 8 \\
- & 1 & 7 & 0 & 0 & 0 \\
\hline
 & 1 & 5 & 4 & 6 & 8 \\
- & 1 & 5 & 3 & 0 & 0 \\
\hline
 & & & 1 & 6 & 8 \\
 & & - & 1 & 5 & 3 \\
\hline
 & & & & 1 & 5
\end{array}
$$

$$
\begin{aligned}
32\,468 = \; & 17 \times \mathbf{1} \times 1000 \\
& +17 \times \mathbf{9} \times 100 \\
& +17 \times \mathbf{0} \times 10 \\
& +17 \times \mathbf{9} \times 1 \\
& +15
\end{aligned}
$$

$$32\,468 = 17 \times (\mathbf{1} \times 1000 + \mathbf{9} \times 100 + \mathbf{0} \times 10 + \mathbf{9} \times 1) + 15$$

Let’s do it again!

$$
\begin{array}{r|rrrrr}
 & \color{red}{1} & \color{red}{9} & \color{red}{0} & \color{red}{9} & \\
\hline
17 & 3 & 2 & 4 & 6 & 8 \\
- & 1 & 7 & 0 & 0 & 0 \\
\hline
 & 1 & 5 & 4 & 6 & 8 \\
- & 1 & 5 & 3 & 0 & 0 \\
\hline
 & & & 1 & 6 & 8 \\
 & & - & 1 & 5 & 3 \\
\hline
 & & & & 1 & 5
\end{array}
$$

$$
\begin{aligned}
32\,468 = \;& 17 \times \mathbf{1} \times 1000 \\
& +17 \times \mathbf{9} \times 100 \\
& +17 \times \mathbf{0} \times 10 \\
& +17 \times \mathbf{9} \times 1 \\
& +15
\end{aligned}
$$

$$32\,468 = \; 17 \times (\mathbf{1}000 + \mathbf{9}00 + \mathbf{0} \times 10 + \mathbf{9}) + 15$$

Let's do it again!

$$
\begin{array}{r|rrrrr}
 & & 1 & 9 & 0 & 9 \\
\hline
17 & 3 & 2 & 4 & 6 & 8 \\
- & 1 & 7 & 0 & 0 & 0 \\
\hline
 & 1 & 5 & 4 & 6 & 8 \\
- & 1 & 5 & 3 & 0 & 0 \\
\hline
 & & & 1 & 6 & 8 \\
 & & - & 1 & 5 & 3 \\
\hline
 & & & & 1 & 5
\end{array}
$$

$$
\begin{aligned}
32\,468 = \; & 17 \times 1 \times 1000 \\
& +17 \times 9 \times 100 \\
& +17 \times 0 \times 10 \\
& +17 \times 9 \times 1 \\
& +15
\end{aligned}
$$

$$32\,468 = \; 17 \times 1909 + 15$$

What does the algorithm actually do?

It’s a series of subtractions!

We subtract the biggest common multiple of the divisor that has as many trailing zeros as possible.

And then we do that at each step until the result is less than the divisor.

The result is the remainder.

		4	7	2	3
13	6	1	4	0	3
−	5	2	0	0	0
		9	4	0	3
		−9	1	0	0
			3	0	3
			−2	6	0
				4	3
				−3	9
					4

What was the point of all this again?

Understand how an algorithm works

When given an algorithm to study, students need to know:

- where to start
- what questions to ask
- how to test their understanding.

They can practise this with algorithms they should already be familiar with.

Why left-to-right?

Smaller numbers are easier!

In addition, subtraction and multiplication, we start with the units and only need to look one place value column to the left at a time to regroup/carry values.

In long division, we subtract the largest possible common multiple of (the largest possible) power of 10 and the divisor which results in a positive value. This means we are subtracting large numbers with lots of zeros (which is easier) – **all to make the result as small as possible!**

$$
\begin{array}{r|rrrrr}
 & & 1 & 9 & 0 & 9 \\
\hline
17 & 3 & 2 & 4 & 6 & 8 \\
- & 1 & 7 & 0 & 0 & 0 \\
\hline
 & 1 & 5 & 4 & 6 & 8 \\
- & 1 & 5 & 3 & 0 & 0 \\
\hline
 & & & 1 & 6 & 8 \\
 & & - & 1 & 5 & 3 \\
\hline
 & & & & 1 & 5
\end{array}
$$

What about short division?

Short division is just long division where you don’t write it all down.

$$158 \div 7$$

$$\begin{array}{r|lll} & 0 & 2 & 2 \quad \text{remainder } 4 \\ \hline 7 & 1 & 5^{1} & 8 \end{array}$$

$$\begin{array}{r|rrr} & 0 & 2 & 2 \\ \hline 7 & 1 & 5 & 8 \\ & -1 & 4 & \\ \hline & & 1 & 8 \\ & & -1 & 4 \\ \hline & & & 4 \end{array}$$

n	$n \times 7$
1	7
2	14
3	21
4	28
5	35
6	42
7	49
8	56
9	63
10	70

How well do we understand the long division algorithm?

Can we work backwards to construct a long division?

What are the key pieces of information we need to calculate a dividend?

- Divisor
- Quotient
- Remainder

How much freedom do we have with these values?

$$
\begin{array}{r|rrrrr}
 & & 1 & 9 & 0 & 9 \\
\hline
17 & 3 & 2 & 4 & 6 & 8 \\
- & 1 & 7 & 0 & 0 & 0 \\
\hline
 & 1 & 5 & 4 & 6 & 8 \\
- & 1 & 5 & 3 & 0 & 0 \\
\hline
 & & & 1 & 6 & 8 \\
 & & - & 1 & 5 & 3 \\
\hline
 & & & & 1 & 5
\end{array}
$$

Working backwards

Can we construct a long division? Let's try by using a random number generator….

Choose a divisor D … 44

$$
\begin{aligned}
N = {} & D \times \boldsymbol{a_3} \times 1000 \\
& + D \times \boldsymbol{a_2} \times 100 \\
& + D \times \boldsymbol{a_1} \times 10 \\
& + D \times \boldsymbol{a_0} \times 1 \\
& + R
\end{aligned}
$$

Working backwards

Can we construct a long division? Let's try by using a random number generator….

Choose values to fill in for a_0, a_1, a_2 and a_3 (to give us the quotient).
What are the upper and lower bounds on these values?

$a_0 = 2$, $a_1 = 4$, $a_2 = 8$ and $a_3 = 2$

$$
\begin{aligned}
N = {} & 44 \times \boldsymbol{a_3} \times 1000 \\
& +44 \times \boldsymbol{a_2} \times 100 \\
& +44 \times \boldsymbol{a_1} \times 10 \\
& +44 \times \boldsymbol{a_0} \times 1 \\
& +R
\end{aligned}
$$

Working backwards

Can we construct a long division? Let's try by using a random number generator….

Choose a value for R.
What are the upper and lower bounds for R? $R = 10$

$$\begin{aligned} N = {} & 44 \times \mathbf{2} \times 1000 \\ & +44 \times \mathbf{4} \times 100 \\ & +44 \times \mathbf{8} \times 10 \\ & +44 \times \mathbf{2} \times 1 \\ & +R \end{aligned}$$

Working backwards

Can we construct a long division?

$$
\begin{aligned}
N &= 44 \times \mathbf{2} \times 1000 \\
&\quad + 44 \times \mathbf{4} \times 100 \\
&\quad + 44 \times \mathbf{8} \times 10 \\
&\quad + 44 \times \mathbf{2} \times 1 \\
&\quad + 10
\end{aligned}
\longrightarrow
\begin{aligned}
N &= 44 \times \mathbf{2}000 \\
&\quad + 44 \times \mathbf{4}00 \\
&\quad + 44 \times \mathbf{8}0 \\
&\quad + 44 \times \mathbf{2} \\
&\quad + 10
\end{aligned}
$$

Working backwards

Can we construct a long division?

$$\begin{aligned} N = {} & 44 \times 2 \times 1000 \\ & +44 \times 4 \times 100 \\ & +44 \times 8 \times 10 \\ & +44 \times 2 \times 1 \\ & +10 \end{aligned} \quad \longrightarrow \quad \begin{aligned} N = {} & 88\,000 \\ & +17\,600 \\ & +3520 \\ & +88 \\ & +10 \end{aligned}$$

Working backwards

$$N = 88\,000 + 17\,600 + 3520 + 88 + 10$$

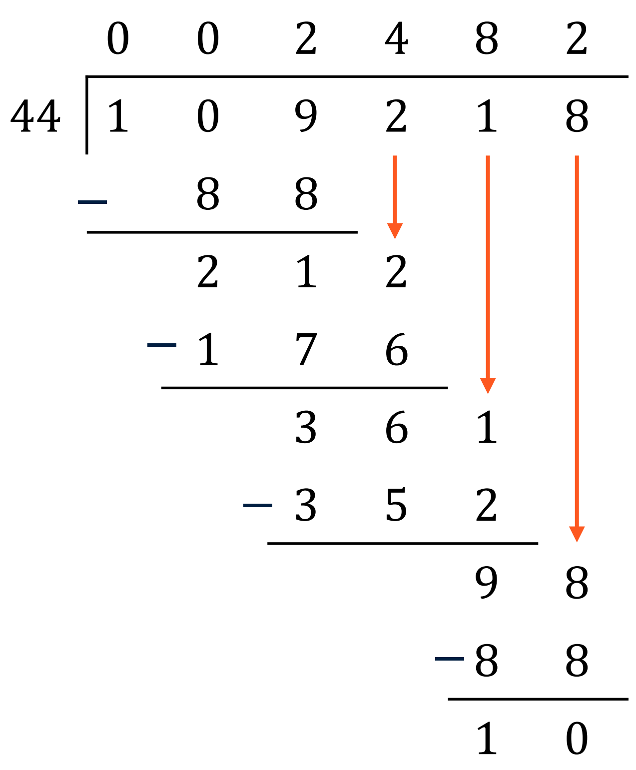
$$\longrightarrow N = 109\,218 \longrightarrow$$

$$109\,218 - 88\,000 - 17\,600 - 3520 - 88 = 10$$

Let's check if the long division matches!

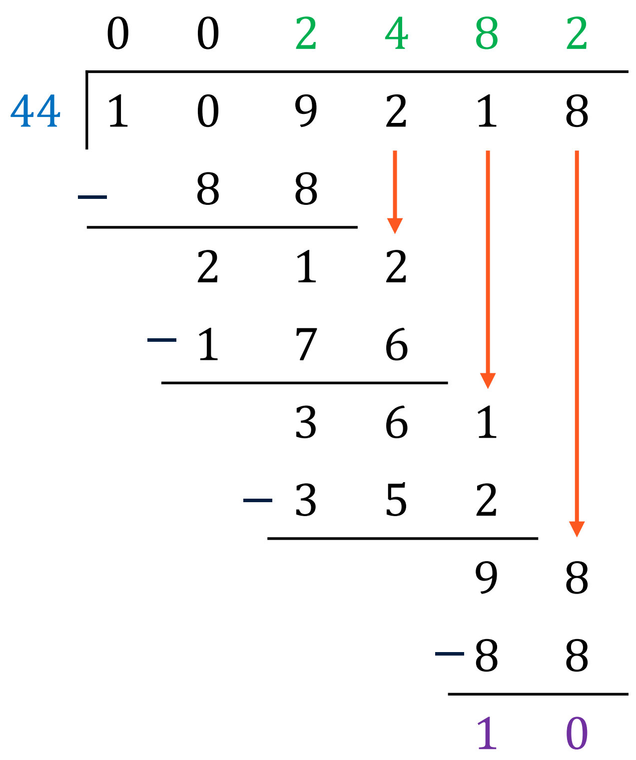
Working backwards

n	$n \times 44$
1	44
2	88
3	132
4	176
5	220
6	264
7	308
8	352
9	396
10	440



Working backwards

n	$n \times 44$
1	44
2	88
3	132
4	176
5	220
6	264
7	308
8	352
9	396
10	440



$$
\begin{aligned}
109\,218 = {} & 44 \times 2 \times 1000 \\
& + 44 \times 4 \times 100 \\
& + 44 \times 8 \times 10 \\
& + 44 \times 2 \times 1 \\
& + 10
\end{aligned}
$$

Working backwards

n	$n \times 44$
1	44
2	88
3	132
4	176
5	220
6	264
7	308
8	352
9	396
10	440

$$
\begin{array}{r|cccccc}
 & 0 & 0 & 2 & 4 & 8 & 2 \\
\hline
44 & 1 & 0 & 9 & 2 & 1 & 8 \\
- & & 8 & 8 & 0 & 0 & 0 \\
\hline
 & & 2 & 1 & 2 & 1 & 8 \\
 & - & 1 & 7 & 6 & 0 & 0 \\
\hline
 & & & 3 & 6 & 1 & 8 \\
 & & - & 3 & 5 & 2 & 0 \\
\hline
 & & & & & 9 & 8 \\
 & & & & - & 8 & 8 \\
\hline
 & & & & & 1 & 0
\end{array}
$$

$$
\begin{aligned}
109\,218 &- 88\,000 \\
&-17\,600 \\
&-3520 \\
&-88 \\
&= 10
\end{aligned}
$$

Polynomials

What about polynomial long division?

$$
\begin{array}{rrrrr}
 & & x^2 & +x & -2 \\
\cline{2-5}
x+1 & \multicolumn{1}{|r}{x^3} & +2x^2 & -x & +3 \\
 & -(x^3 & +x^2) & & \\
\cline{2-3}
 & & x^2 & -x & \\
 & & -(x^2 & +x) & \\
\cline{3-4}
 & & & -2x & +3 \\
 & & & -(-2x & -2) \\
\cline{4-5}
 & & & & 5
\end{array}
$$

Numbers: subtract the largest possible common multiple of the highest possible power of 10 and the divisor which results in a positive value.

What is the equivalent for polynomials?

Polynomials

What about polynomial long division?

$$
\begin{array}{r|llll}
 & x^2 & +x & -2 & \\
\hline
x+1 & x^3 & +2x^2 & -x & +3 \\
 & -(x^3 & +x^2) & \downarrow & \downarrow \\
\cline{2-3}
 & & x^2 & -x & \\
 & & -(x^2 & +x) & \\
\cline{3-4}
 & & & -2x & +3 \\
 & & & -(-2x & -2) \\
\cline{4-5}
 & & & & 5
\end{array}
$$

Polynomials: subtract the common multiple of the highest possible power of x and the divisor which results in a polynomial with a strictly lower* degree.

*We need to think carefully about the degree of the zero polynomial.

Polynomials

What else can we do?

Let's try what we did before…

$$
\begin{array}{r|rrrr}
 & x^2 & +x & -2 & \\
\hline
x+1 & x^3 & +2x^2 & -x & +3 \\
 & -(x^3 & +x^2 & +0x & +0) \\
\hline
 & & x^2 & -x & +3 \\
 & & -(x^2 & +x & +0) \\
\hline
 & & & -2x & +3 \\
 & & & -(-2x & -2) \\
\hline
 & & & & 5
\end{array}
$$

$$
\begin{aligned}
(x^3 + 2x^2 - x + 3) &- (x^3 + x^2) \\
&-(x^2 + x) \\
&-(-2x - 2) \\
&= 5
\end{aligned}
$$

$$
\begin{aligned}
(x^3 + 2x^2 - x + 3) = &(x^3 + x^2) \\
&+(x^2 + x) \\
&+(-2x - 2) \\
&+5
\end{aligned}
$$

$$
\begin{aligned}
(x^3 + 2x^2 - x + 3) = &(x+1) \times 1 \times x^2 \\
&+(x+1) \times 1 \times x \\
&+(x+1) \times (-2) \times 1 \\
&+5
\end{aligned}
$$

$$(x^3 + 2x^2 - x + 3) = (x+1)(1 \times x^2 + 1 \times x + (-2) \times 1) + 5$$

$$(x^3 + 2x^2 - x + 3) = (x+1)(x^2 + x - 2) + 5$$

Polynomials

What else can we do?

Let's try what we did before…

$$
\begin{array}{r|rrrr}
 & 1x^2 & +1x & -2 & \\
\hline
x+1 & x^3 & +2x^2 & -x & +3 \\
 & -(x^3 & +x^2 & +0x & +0) \\
\hline
 & & x^2 & -x & +3 \\
 & & -(x^2 & +x & +0) \\
\hline
 & & & -2x & +3 \\
 & & & -(-2x & -2) \\
\hline
 & & & & 5
\end{array}
$$

$$
\begin{aligned}
(x^3 + 2x^2 - x + 3) &- (x^3 + x^2) \\
&-(x^2 + x) \\
&-(-2x - 2) \\
&= 5
\end{aligned}
$$

$$
\begin{aligned}
(x^3 + 2x^2 - x + 3) = &(x^3 + x^2) \\
&+(x^2 + x) \\
&+(-2x - 2) \\
&+5
\end{aligned}
$$

$$
\begin{aligned}
(x^3 + 2x^2 - x + 3) = &(x + 1) \times 1 \times x^2 \\
&+(x + 1) \times 1 \times x \\
&+(x + 1) \times (-2) \times 1 \\
&+5
\end{aligned}
$$

$$(x^3 + 2x^2 - x + 3) = (x + 1)(1 \times x^2 + 1 \times x + (-2) \times 1) + 5$$

$$(x^3 + 2x^2 - x + 3) = (x + 1)(1x^2 + 1x - 2) + 5$$

Polynomials

What else could we try? Let's substitute $x = 10$.

$$
\begin{array}{r|rrrr}
 & x^2 & +x & -2 & \\
\hline
x+1 & x^3 & +2x^2 & -x & +3 \\
 & -(x^3 & +x^2) & & \\
\cline{2-3}
 & & x^2 & -x & \\
 & & -(x^2 & +x) & \\
\cline{3-4}
 & & & -2x & +3 \\
 & & & -(-2x & -2) \\
\cline{4-5}
 & & & & 5
\end{array}
\qquad
\begin{array}{r|rrrr}
 & 10^2 & +10 & -2 & \\
\hline
10+1 & 10^3 & +2\times 10^2 & -10 & +3 \\
 & -(10^3 & +10^2) & & \\
\cline{2-3}
 & & 10^2 & -10 & \\
 & & -(10^2 & +10) & \\
\cline{3-4}
 & & & -2\times 10 & +3 \\
 & & & -(-2\times 10 & -2) \\
\cline{4-5}
 & & & & 5
\end{array}
$$

Polynomials

$$
\begin{array}{r|llll}
 & x^2 & +x & -2 & \\
\hline
x+1 & x^3 & +2x^2 & -x & +3 \\
 & -(x^3 & +x^2) & \downarrow & \downarrow \\
 & & x^2 & -x & \\
 & & -(x^2 & +x) & \downarrow \\
 & & & -2x & +3 \\
 & & & -(-2x & -2) \\
 & & & & 5
\end{array}
$$

$$
\begin{array}{r|llll}
 & 100 & +10 & -2 & \\
\hline
11 & 1000 & +200 & -10 & +3 \\
 & -(1000 & +100) & \downarrow & \downarrow \\
 & & 100 & -10 & \\
 & & -(100 & +10) & \downarrow \\
 & & & -20 & +3 \\
 & & & -(-20 & -2) \\
 & & & & 5
\end{array}
$$

Let’s compare this with the usual long division.

Polynomials

$$
\begin{array}{r|rrrr}
 & 100 & +10 & -2 & \\
\hline
11 & 1000 & +200 & -10 & +3 \\
 -(& 1000 & +100) & & \\
\hline
 & & 100 & -10 & \\
 & -(& 100 & +10) & \\
\hline
 & & & -20 & +3 \\
 & & -(& -20 & -2) \\
\hline
 & & & & 5
\end{array}
$$

$$
\begin{array}{r|rrrr}
 & & 1 & 0 & 8 \\
\hline
11 & 1 & 1 & 9 & 3 \\
-(& 1 & 1) & & \\
\hline
 & & 0 & 9 & 3 \\
 & & -(& 8 & 8) \\
\hline
 & & & & 5
\end{array}
$$

Further questions about polynomials

1. Can we 'construct' a polynomial long division?

2. Are there any restrictions on the green numbers (coefficients)?

3. What happens if you restrict to integer coefficients from $0 - 9$ and then substitute $x = 10$? What does this look like?

4. How many 'long divisions' are there for the same division calculation?

Key takeaways

Students need to understand how algorithms work.

We need to give them the skills to study an algorithm and work out how it works.

There are many algorithms in maths – pick one and see what you can do.